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The Pauli form factor of the quark induced by instantons

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Abstract

The non-perturbative contribution to the Pauli form factor of the quark, $F_2(Q^2)$, is calculated within an instanton model for the QCD vacuum. It is shown that the instantons give a large negative contribution to the form factor.

The interaction of photons with hadrons is one of the most powerful tools to investigate the structure of strong interactions. The electromagnetic form factors of hadrons are now widely discussed [1]. This increased interest in the electromagnetic form factors is related to the significant theoretical progress in understanding the connection between form factors and structure functions of hadrons through the generalized parton distributions (GPD) [2], and to the need to explain the new experimental data from Jefferson Lab [3] for the nucleon and pion form factors at large Q^2. One of the basic components of the calculation of the form factors within the constituent quark model are the electromagnetic form factors of quarks. It has been shown that the non-elementarity of the constituent quarks plays a crucial role for the understanding of the parton distributions and form factors [4,5]. Moreover, quarks form factors have been instrumental in the investigation of the scaling violations in deep-inelastic scattering [6]. We should also mention that in order to explain the high twist effects in Drell–Yan processes [7], various perturbative and non-perturbative gluonic corrections to the photon–quark vertex must be taken into account [8].

The general formula for the photon–quark vertex is

$$\Gamma_\mu = F_1(Q^2)\gamma_\mu + \frac{iq_\nu\sigma_{\mu\nu}}{2m_q}F_2(Q^2), \tag{1}$$

where m_q is the mass of quark, $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$ and $F_1(Q^2)$, $F_2(Q^2)$ are the Dirac and Pauli form factors, correspondingly. There exist calculations, both perturbative and non-perturbative, of QCD contributions to the $F_1(Q^2)$ form factor (see [9,10] and references therein).

The usual way to incorporate the non-perturbative QCD dynamics into the calculation of quark form factors is the consideration of the renormalon contributions [10]. However, renormalon contributions amount only to a part of the QCD non-perturbative effects.

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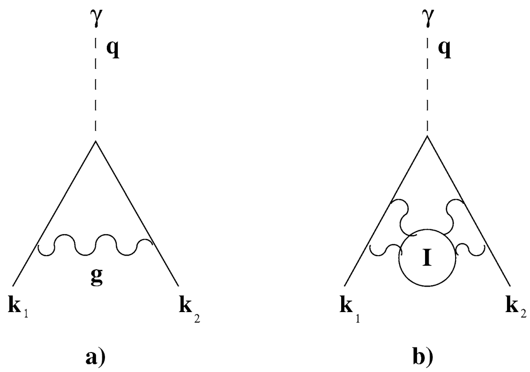

Fig. 1. The contribution to the quark form factor $F_2(Q^2)$ from (a) one gluon exchange and (b) instanton.

The instanton model for the QCD vacuum is now one of the most successful models for the description of non-perturbative effects in strong interactions (see the review [11]). Within this model, the QCD vacuum contains complex topological structures related to existence of the instantons, strong fluctuations of the gluon vacuum fields. In QCD the instantons play an important role in describing the realization of chiral symmetry breaking and the origin of the masses of constituent quarks and hadrons. In Refs. [12–17] it was shown that the instantons produce also specific non-perturbative effects in various hadronic reactions. Recently, the first step in the calculation of instanton contributions to the quark form factor was done in Ref. [19]. Within the Wilson integral approach, it was shown that the instantons lead to a finite renormalization of the next-to-leading perturbative correction to the $F_1(Q^2)$ form factor.

One of the most important features of the instanton induced quark–quark and quark–gluon interactions, in comparison with the perturbative gluon exchange, is the possibility of having quark chirality-flip due to the instanton field. From this point of view, the chirality-flip $F_2(Q^2)$ quark form factor plays the role of a filter of instantons, since the perturbative contribution to this form factor at large $Q^2 \gg m_q^2$, is small, i.e., proportional to m_q^2/Q^2 [20]. Therefore, the dominance of the instanton contribution to $F_2(Q^2)$ for the light u-, d-, s-quarks is expected even at large momentum transfers.

At small Q^2, the Pauli form factor receives also contributions from both, perturbative gluon exchanges and non-perturbative QCD effects [21]. It is well known that the perturbative Schwinger correction [22] (Fig. 1(a)) to the quark anomalous magnetic moment $\kappa_q = F_2(0)$ is rather large and *positive*

$$\kappa_q^{\mathrm{pQCD}} = \frac{2\alpha_s(m_q^2)}{3\pi} \approx 0.14, \tag{2}$$

where the so-called analytic running constant [23]

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0}\left[\frac{1}{\log(Q^2/\Lambda^2)} + \frac{\Lambda^2}{\Lambda^2 - Q^2}\right] \tag{3}$$

has been used with $\Lambda \approx 300$ MeV and we have used $m_q \approx 350$ MeV for the constituent mass of the quark in the hadron, which corresponds to the magnitude of the quark effective mass in the instanton liquid model [24]. Such large contribution would result in a modification of the constituent quark predictions for the electromagnetic properties of hadrons, which, however, describe rather well the data without a sizeable quark anomalous magnetic moment (see discussion in [21]). Furthermore, from the analysis of the scaling violations observed in deep-inelastic scattering at Jefferson Lab, performed within the constituent quark model [6], a *negative* value of $F_2(Q^2)$ form factor at low Q^2 has been obtained. It is therefore very difficult to explain the behaviour of $F_2(Q^2)$ using only the perturbative QCD framework.

In this Letter we calculate the instanton contribution to the $F_2(Q^2)$ form factor of the quark (Fig. 1(b)).

It is convenient to use the two component Dirac spinors

$$\Psi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}, \tag{4}$$

and the following representation for γ matrices:

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \tag{5}$$

where, in Minkowski space–time, $\sigma_\mu = (1, \vec{\sigma}_\mu)$, $\bar{\sigma}_\mu = (1, -\vec{\sigma}_\mu)$, and in Euclidean space–time $\sigma_\mu = (-i\vec{\sigma}_\mu, 1)$, $\sigma_\mu = (i\vec{\sigma}_\mu, 1)$. The photon–quark vertex which corresponds to the transition of the incoming left-hand quark to the outgoing right-hand quark in Fig. 1(b), is

$$A_\mu^{LR} = F_2\big(Q^2\big)\frac{q_\nu \chi_R^+(\sigma^\nu\bar{\sigma}^\mu - \sigma^\mu\bar{\sigma}^\nu)\chi_L}{4m_q}. \tag{6}$$

As was discovered by 't Hooft [18] there is a zero mode for the quark in the instanton field. This mode has the chirality required to produce a helicity flip of scattered quark. The non-zero contribution to A_μ^{LR} arises when either the incoming, or the outgoing quarks in Fig. 1(b) scatter through the zero-mode of the instanton field. The result of calculation is

$$A_\mu^{LR,\mathrm{inst}} = -i\int dU\, d\rho\, \frac{d(\rho)}{\rho^5}\, \chi^+(k_2)(ik_2)\big[\Phi_0(-k_2)V_\mu(q,k_1) + \bar{V}_\mu(q,-k_2)\Phi_0^+(ik_1)\big](ik_1)\chi_L(k_1)/m_q, \tag{7}$$

where

$$V_\mu(q,k_1) = \int d^4x\, e^{-iqx}\Phi(x)^+\bar{\sigma}_\mu S_{nz}^{(I)}(x,k_1), \qquad \bar{V}_\mu(q,-k_2) = \int d^4x\, e^{-iqx}\bar{S}_{nz}^{(I)}(-k_2,x)\sigma_\mu\Phi(x), \tag{8}$$

where Φ_0 (Φ_0^+) and $S_{nz}^{(I)}$, $\bar{S}_{nz}^{(I)}$ are the zero-mode and non-zero mode quark propagators for the incoming (outgoing) quarks. In Eq. (7), ρ is the instanton size, $d(\rho)$ is the instanton density, dU stands for the integration over instanton orientation in colour space, and $k \equiv k_\mu\sigma^\mu$, $\bar{k} \equiv k_\mu\bar{\sigma}^\mu$ for any four-vector k_μ [13]. In Eq. (7) the continuation from Euclidean space–time to Minkowski space–time should be done by substituting $k_{i4} \to (-i+\epsilon)\sqrt{\vec{k}_i^2}$, $q_4 \to (-i+\epsilon)q_0$.

For the light quarks $k_{1,2}^2 = m_q^2 \approx 0$ the zero-mode is well known asymptotically

$$\big((ik_2)\Phi_0(-k_2)\big)^{\alpha i} \quad\to\quad 2\pi\rho U_\beta^i\epsilon^{\beta\alpha}, \qquad \big(\Phi_0(k_1)^+(i\bar{k}_1)\big)^{\alpha i} \quad\to\quad 2\pi\rho\epsilon^{\beta\alpha}\big(U^+\big)_\beta^i, \tag{9}$$

where U is the orientation matrix of the instanton in colour space. The asymptotic behaviour of the non-zero mode propagators at $Q^2 \gg m_q^2$ was found recently in Ref. [13]

$$\begin{aligned} S_{nz}^{(I)}(x,k_1)(ik_1) \quad &\to\quad -\frac{e^{-ik_1x}}{\sqrt{\Pi(x)}}\left(1 + \big(1-e^{ik_1x}\big)\frac{\rho^2}{x^2}\frac{Ux\bar{k}_1U}{2k_1x}\right), \\ (ik_2)\bar{S}_{nz}^{(I)}(-k_2,x) \quad &\to\quad -\frac{e^{ik_2x}}{\sqrt{\Pi(x)}}\left(1 + \big(1-e^{-ik_2x}\big)\frac{\rho^2}{x^2}\frac{Uk_2\bar{x}U}{2k_2x}\right), \end{aligned} \tag{10}$$

where $\Pi(x) = 1 + \rho^2/x^2$. By using these formulas, after integration over the colour orientations (see [11,13]) and a simple algebra, we get

$$A_\mu^{LR,\mathrm{inst}} = \frac{4\pi^2}{3Q^2m_q}\int d\rho\, \frac{d(\rho)}{\rho^3}\big(\rho QK_1(\rho Q) - 1\big)q_\nu\chi_R^+\big(\sigma^\nu\bar{\sigma}^\mu - \sigma^\mu\bar{\sigma}^\nu\big)\chi_L. \tag{11}$$

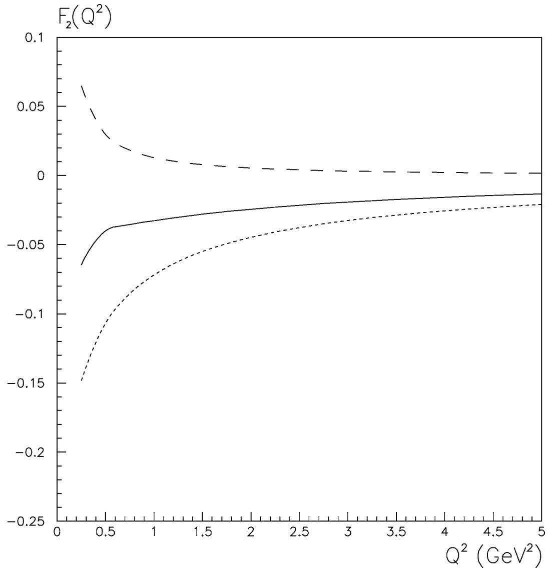

Fig. 2. The contributions to the Pauli form factor of the quark from (a) instantons (short-dashed line), (b) perturbative gluon exchanges (long-dashed line) and (c) their sum (solid line).

From (11) and (6), the following result is obtained for the instanton contribution to the form factor:

$$F_2^{\text{inst}}\big(Q^2\big) = \frac{16\pi^2}{3Q^2}\int \frac{d\rho\, d(\rho)}{\rho^3}\big(\rho Q K_1(\rho Q) - 1\big). \tag{12}$$

For a numerical estimate of this contribution, we use the instanton liquid model suggested by Shuryak [25]. Within this model, the density of instantons is

$$\frac{d(\rho)}{\rho^5} = n_I\delta(\rho - \rho_c), \tag{13}$$

where $n_I \approx 1/2\ \text{fm}^{-4}$, $\rho_c \approx 1/3$ fm are the average density and size of instantons in the QCD vacuum, correspondingly. The final result for the form factor is

$$F_2^{\text{inst}}\big(Q^2\big) = \frac{8f}{3}\frac{(\rho_c Q K_1(\rho_c Q) - 1)}{\rho_c^2 Q^2}, \tag{14}$$

where $f = n_I\pi^2\rho_c^4 \approx 0.1$ is the so-called packing fraction (diluteness) of the instantons in the vacuum. In Fig. 2 the contribution to the quark Pauli form factor arising from instantons (14) is compared with the result of the double-logarithmic perturbative calculation [20]

$$F_2\big(Q^2\big)^{\text{pQCD}} = \frac{\alpha_s(Q^2)C_F}{\pi}\frac{m_q^2\ln(Q^2/m_q^2)}{Q^2}\exp\left(-\frac{\alpha_s(Q^2)C_F}{4\pi}\ln^2\left(\frac{Q^2}{m_q^2}\right)\right), \tag{15}$$

at $Q^2 = 0.25$–$5\ \text{GeV}^2$ for the quark mass[1] $m_q \approx 350$ MeV and analytic running coupling constant $\alpha_s(Q^2)$ given in (3).

[1] For the current mass of the quark $m_q \approx 5$ MeV, the pQCD contribution to F_2 form factor is very tiny (10^{-5}–10^{-9}) in this interval of Q^2.

It is evident that the instantons give a large negative contribution to the quark form factor in the region of momentum transfer of order of a few GeV^2. This contribution is larger then the positive contribution coming from the perturbative exchange, and one may expect that the total Pauli form factor of the quark should be negative.

Therefore the instanton corrections to the chirality-flip quark–photon vertex cannot be neglected in the photon induced reactions in this kinematic region. The application of the obtained result for the specific reactions will be the subject of forthcoming publications.

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